

Name _____

Model Integer Addition

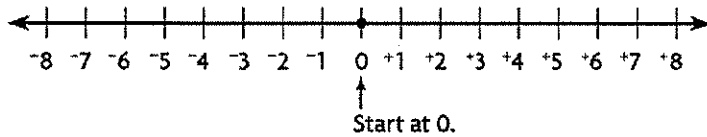
You can use a number line to model the addition of integers.

Move **right** on the number line to add a **positive** integer.
Move **left** on the number line to add a **negative** integer.

Use a number line to find $-3 + +5$.

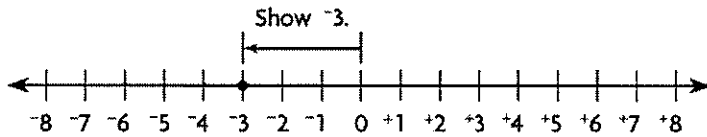
Step 1

Draw a number line.
Start at 0.



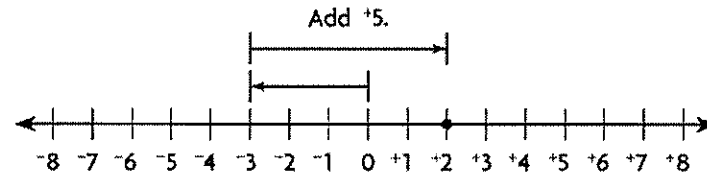
Step 2

Show -3 .
Move left 3 units.



Step 3

Add $+5$.
It is **positive**, so move **right**
5 units from -3 .
Read the integer where
you stop: $+2$.
So, $-3 + +5 = +2$.



Draw a number line to find the sum.

1. $-4 + +1$ _____

2. $-5 + -2$ _____

3. $+2 + -2$ _____

4. $-3 + -5$ _____

5. $-6 + +8$ _____

6. $+7 + -4$ _____

Name _____

Model Integer Subtraction

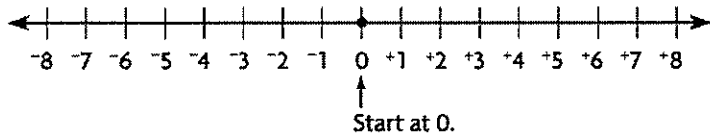
You can use a number line to model the subtraction of integers.

Move **left** on the number line to subtract a **positive** integer.
Move **right** on the number line to subtract a **negative** integer.

Use a number line to find $-1 - +3$.

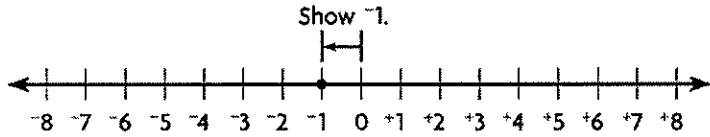
Step 1

Draw a number line.
Start at 0.



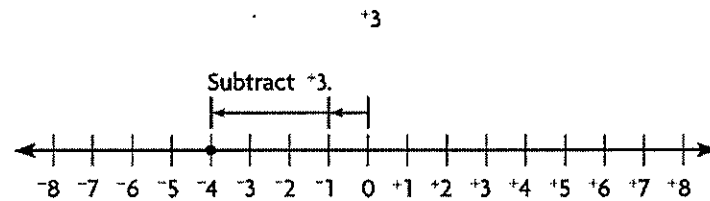
Step 2

Show -1 .
Move left 1 unit.



Step 3

Subtract $+3$.
It is **positive**, so move **left**.
From -1 , move left 3 units.
Read the integer where you stop: -4 .



So, $-1 - +3 = -4$.

Draw a number line to find the difference.

- | | | |
|--------------------|--------------------|--------------------|
| 1. $-3 - +2$ _____ | 2. $-5 - -5$ _____ | 3. $+1 - -6$ _____ |
| 4. $-3 - -5$ _____ | 5. $-3 - +3$ _____ | 6. $+5 - +8$ _____ |

Name _____

Model Integer Multiplication

To model integer multiplication, you can use two-color counters or number lines.

Multiply $-3 \times +5$.

Step 1 Use the Commutative Property, if necessary, to rewrite the expression so the positive factor is first. $-3 \times +5 = +5 \times (-3)$

Step 2 Make groups of black counters. 5 groups of 3 black counters is 15 black counters so $-3 \times +5 = -15$.

Counters



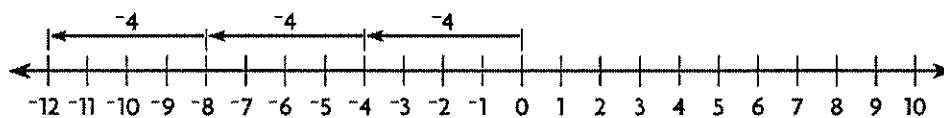
Number Line

Multiply $-4 \times +3$.

Step 1 Use the Commutative Property, if necessary, to write the positive factor first. $-4 \times +3 = +3 \times (-4)$

Step 2 Start at zero. The first factor tells how many times to jump. The second factor tells how many units long and in what direction to make each jump.

Step 3 The product is the number on which you land. $-4 \times +3 = -12$



Use counters or a number line to find each product.

1. $-2 \times +5 =$ _____ 2. $8 \times (-2) =$ _____ 3. $+9 \times (-1) =$ _____ 4. $-6 \times 5 =$ _____

Name _____

Simplify Complex Fractions

A **complex fraction** is a fraction in which the numerator, denominator, or both contain fractions. For example, $\frac{\frac{3}{4}}{2}$, $\frac{1}{\frac{5}{6}}$, and $\frac{\frac{4}{9}}{\frac{3}{2}}$ are all complex fractions.

Simplify $\frac{\frac{5}{12}}{\frac{15}{24}}$. Write the answer in simplest form.

Step 1 Write the fraction as a division problem. $\frac{5}{12} \div \frac{15}{24}$

Step 2 Multiply the first fraction by the reciprocal of the second fraction. $\frac{5}{12} \times \frac{24}{15}$

Simplify first. Then multiply. $\frac{\cancel{5}^1}{\cancel{12}_3} \times \frac{24^2}{\cancel{15}_3} = \frac{1}{1} \times \frac{2}{3} = \frac{2}{3}$

So, the quotient is $\frac{2}{3}$.

Simplify. Write the answer in simplest form.

1. $\frac{\frac{4}{9}}{\frac{16}{27}}$ Division problem: _____

2. $\frac{\frac{20}{27}}{5}$

Quotient: _____

3. $\frac{\frac{1}{8}}{\frac{3}{26}}$

4. $\frac{\frac{6}{7}}{\frac{18}{7}}$

Name _____

Identify Proportional Relationships

A **proportional relationship** is a relationship between two quantities in which the ratio of one quantity to the other is constant.

A human fingernail grows 3 centimeters per year. Is the relationship between the amount of growth and the number of years a proportional relationship?

Step 1 Make a table of values.

| | | | | |
|-----------------------|---|---|---|----|
| Number of years | 1 | 2 | 3 | 4 |
| Amount of growth (cm) | 3 | 6 | 9 | 12 |

\downarrow \downarrow \downarrow \downarrow

Step 2 Find and compare ratios.

$$\frac{\text{amount of growth}}{\text{number of years}} = \frac{3}{1} = \frac{6}{2} = \frac{9}{3} = \frac{12}{4}$$

The ratios are constant, so the relationship is a proportional relationship.

1. Brian's age is 6 years greater than Amy's age. Is the relationship between Brian's age and Amy's age a proportional relationship?
2. The number of legs is 8 times the number of spiders. Is the relationship between the number of legs and the number of spiders a proportional relationship?

a. Complete the table.

| | | | | |
|-------------|---|---|---|---|
| Amy's age | 1 | 2 | 3 | 4 |
| Brian's age | | | | |

b. Find the ratios.

c. Tell whether the relationship is a proportional relationship.

a. Complete the table.

| | | | | |
|---------|---|---|---|---|
| Spiders | 1 | 2 | 3 | 4 |
| Legs | | | | |

b. Find the ratios.

c. Tell whether the relationship is a proportional relationship.

Name _____

Analyze Proportional Relationships

A **proportional relationship** is a relationship between two variables, x and y , that can be written as $y = kx$, where k is a nonzero number called the **constant of proportionality**.

Carpet costs \$12 per square yard. Write and graph an equation that represents this situation. Give the constant of proportionality.

Step 1 Write an equation that relates x and y .
Let x represent the number of square yards of carpet. Let y represent the cost.

The cost is \$12 times the number of square yards.

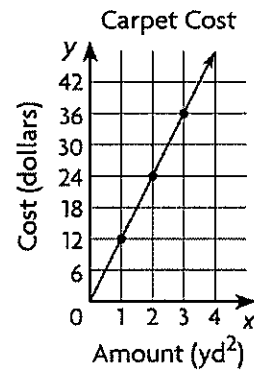
$$y = 12 \cdot x$$

Step 2 Graph the relationship by plotting several points and drawing a line through the origin.

Step 3 Identify the constant of proportionality.

| x | y |
|-----|-----|
| 1 | 12 |
| 2 | 24 |
| 3 | 36 |

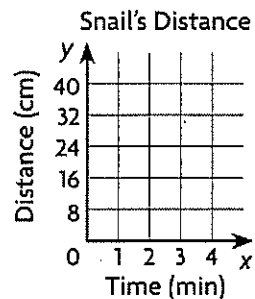
The constant of proportionality is 12.



A snail travels 16 centimeters per minute.

- Write an equation that relates x and y . Let x represent the number of minutes. Let y represent the number of centimeters the snail travels.

- Graph the equation and give the constant of proportionality.



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Apply Percents

Discount and **sales tax** are very common terms involving money and shopping. They both involve percents, because the amount of discount or tax depends on the original price. A discount is subtracted from the original price, and sales tax is added to the original price.

A chicken salad costs \$8.50. The sales tax is 5%. What is the final cost?

Step 1 Find 5% of \$8.60. This will give the amount of sales tax. $5\% \text{ of } \$8.60 = \frac{5}{100} \times 8.60 = 0.43$
The sales tax is \$0.43.

Step 2 Add to find the total cost. price + sales tax = final cost
 $\$8.60 + \$0.43 = \$9.03$
So, the final cost of the chicken salad is \$9.03.

Find the discount and sale price, or the sales tax and final cost.

1. regular price: \$260

discount: 20%

discount: _____

sale price: _____

2. regular price: \$175

sales tax: 6%

sales tax: _____

final cost: _____

3. regular price: \$55.60

sales tax: 5%

sales tax: _____

final cost: _____

4. regular price: \$765

discount: 15%

discount: _____

sale price: _____

Name _____

Percent of Change

You can use percents to describe a change. The **percent of change** is the ratio of the amount of change to the original amount. **Percent of increase** describes an amount that has gone up. **Percent of decrease** describes an amount that has gone down.

$$\text{percent of change} = \frac{\text{amount of change}}{\text{original amount}}$$

Find the percent of change when 15 is increased to 18.

Step 1 Notice that the change is an increase. Find the amount of change.

$$\text{amount of increase} = 18 - 15 = 3$$

Step 2 Find the percent of increase.

$$\begin{aligned} \text{percent of change} &= \frac{\text{amount of change}}{\text{original amount}} \\ &= \frac{3}{15} \\ &= 0.2 = 20\% \end{aligned}$$

Write the formula.

Substitute.

Divide. Write the quotient as a percent.

So, the percent of change is a 20% increase.

Find the percent of change. Label the change as increase or decrease.

1. 10 is increased to 13.

2. 200 is increased to 204.

3. 25 is decreased to 15.

4. 50 is decreased to 11.

5. 124 is increased to 155.

6. 64 is decreased to 56.

7. 27 is increased to 54.

8. 100 is decreased to 1.

9. 55 is increased to 550.

Name _____

Add Algebraic Expressions

Algebraic expressions use numbers, variables, and operation symbols. The expression $5 + 4y$ shows that the two terms, 5 and $4y$, are being added. You can use the Commutative and Associative properties to add expressions.

Find the sum of $(9 + 4w)$ and $(10 + 12w)$.

Step 1 Write the algebraic expression for the sum. $(9 + 4w) + (10 + 12w)$

Step 2 The Associative Property of Addition says you can change the grouping of addends. Write the expression without parentheses. $9 + 4w + 10 + 12w$

Step 3 The Commutative Property of Addition says you can change the order of addends. $9 + 10 + 4w + 12w$

Step 4 Group like terms, and then add like terms. $19 + 16w$

Find the sum of the expressions.

1. $(b + 4) + (13 + 2b)$

2. $(15q + 8) + (9q + 0)$

3. $(26 + x) + (10 + 6x)$

4. $(11 + 15f) + (11f + 9)$

5. $(66e + 18) + (2e + 6)$

6. $(100v + 7) + (15 + v)$

Name _____

Solve Two-Step Equations

Stan had half a pitcher of juice. After he poured 2 cups of juice, there were 6 cups left in the pitcher. How many cups of juice are in a whole pitcher?

Use the equation $\frac{1}{2}c - 2 = 6$ to solve the problem.

Step 1 Write the equation. Notice there is more than one operation involved.

$\frac{1}{2}c - 2 = 6$

Step 2 Reverse the order of the operations. First undo addition or subtraction. Then undo multiplication or division.

Remember: Do the same thing to both sides of an equation.

Addition is the inverse of subtraction. Add 2 to both sides.

$\frac{1}{2}c - 2 + 2 = 6 + 2$

Division is the inverse of multiplication. Divide both sides by $\frac{1}{2}$. To divide by $\frac{1}{2}$, multiply by the reciprocal.

$2 \cdot \frac{1}{2}c = 8 + 2$

So, a full pitcher has 16 cups of juice.

$c = 16$

1. Explain how to solve $\frac{1}{4}n + 5 = 9$ for n . _____

Find the sum of the expressions.

2. $5 + 3m = 65$

3. $\frac{y}{5} - 1 = 4$

4. $22x + 8 = 8$

5. $8 + \frac{3}{8}k = 20$

Name _____

Solve Inequalities

Inequalities are used when many solutions are involved. Inequalities are like equations, except that they use inequality symbols instead of equal signs.

Mr. Jensen needs to pay a babysitter \$12 per hour. He only has \$48 to spend. For how many hours can he hire the babysitter? Let h = number of hours. Solve the inequality $12h \leq 48$, and explain what the solution means.

Step 1 Write the inequality.

$$12h \leq 48$$

Step 2 Use the Properties of Inequality and inverse operations to get h by itself on one side.

$$\frac{12h}{12} \leq \frac{48}{12}$$

Divide both sides by 12.

Step 3 Explain the solution.

$$h \leq 4$$

The solution $h \leq 4$ means that Mr. Jensen can hire the babysitter for 4 hours or less.

Solve the inequality.

1. $n - 10 > 7$

2. $11a \geq 121$

3. $\frac{c}{3} \leq 12$

4. $16 + q \leq 21$

5. $25c > 150$

6. $h - 1 < 0$

7. $\frac{u}{4} \leq 16$

8. $p + 19 \geq 30$

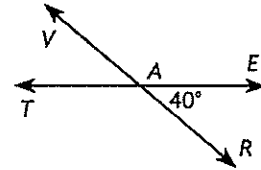
9. $20n < 2,000$

Name _____

Vertical Angles

When two lines intersect, four angles are formed.

- The angles opposite each other are called **vertical angles**. The angle measures of vertical angles are equal.
- The angles next to each other form a straight angle, and the sum of their angle measures is 180° .



Find the measure of $\angle VAT$, $\angle VAE$, and $\angle TAR$.

Step 1 Identify the vertical angles, or angles that are opposite each other.
 $\angle EAR$ and $\angle VAT$
 $\angle VAE$ and $\angle TAR$

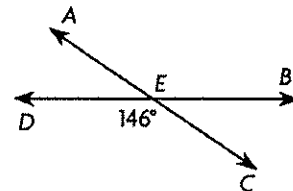
Step 2 Find $m\angle VAT$.
 $m\angle VAT = m\angle EAR$
because they are vertical angles.
 $m\angle EAR = 40^\circ$,
so $m\angle VAT = 40^\circ$.

Step 3 Find $m\angle VAE$.
 $m\angle VAE + m\angle EAR = 180^\circ$
because they form a straight angle.
 $m\angle VAE = 180^\circ - m\angle EAR$
 $m\angle VAE = 180^\circ - 40^\circ$
 $m\angle VAE = 140^\circ$

Step 4 Find $m\angle TAR$.
 $m\angle TAR = m\angle VAE$
because they are vertical angles.
 $m\angle VAE = 140^\circ$,
so $m\angle TAR = 140^\circ$.

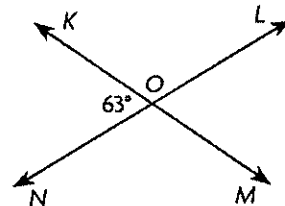
For 1–3, use the drawing to find the measure of the angle.

- $m\angle AED =$ _____
- $m\angle AEB =$ _____
- $m\angle BEC =$ _____



For 4–6, use the drawing to find the measure of the angle.

- $m\angle LOM =$ _____
- $m\angle KOL =$ _____
- $m\angle NOM =$ _____



Name _____

Complementary and Supplementary Angles

Angles that have the same measure are **congruent**.

Pairs of angles whose measures sum to 90° are **complementary angles**.

Pairs of angles whose measures sum to 180° are **supplementary angles**.

An angle cannot be complementary or supplementary by itself; these terms describe a *pair* of angles.

Name a pair of supplementary angles.

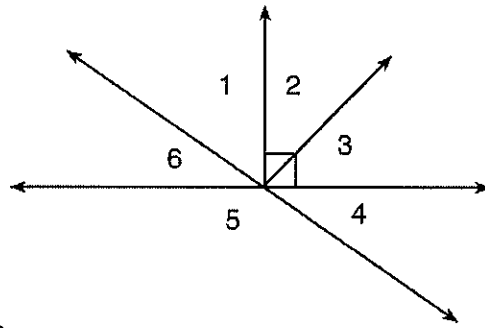
- Look for two adjacent angles that form a straight angle.
- Name the supplementary angles.
The angles are $\angle 4$ and $\angle 5$.
The angles are $\angle 5$ and $\angle 6$.
- If $m\angle 4 = 42^\circ$, what is $m\angle 5$?
Since $\angle 4$ and $\angle 5$ are supplementary, they have a sum of 180. Write and solve an equation.

$$42 + x = 180$$

$$42 - 42 + x = 180 - 42$$

$$x = 138$$

$$m\angle 5 = 138^\circ$$

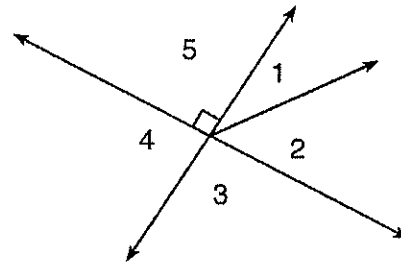


For 1–4, use the figure.

1. Name a pair of supplementary angles.

2. Name a pair of complementary angles.

3. If $m\angle 1 = 25^\circ$, what is $m\angle 2$?



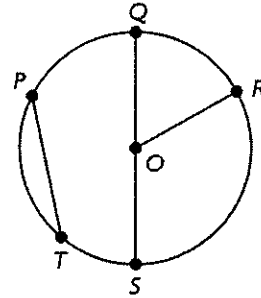
4. Is the sum of the measures of $\angle 2$ and $\angle 3$ less than, equal to, or greater than 180° ? Explain.

Name _____

Parts of a Circle

A **circle** is a closed plane figure with all points on the figure the same distance from a point called the **center** of the circle. Circles are named by the center point. Some other parts of a circle are described in the table below.

| Part of Circle | Description | Example(s) (using Circle <i>D</i>) |
|----------------|---|---|
| radius | a line segment with one endpoint at the center and one endpoint on the circle | \overline{RO} , \overline{QO} , \overline{SO} |
| chord | a line segment whose two endpoints are on the circle | \overline{PT} , \overline{QS} |
| diameter | a chord that passes through the center of the circle | \overline{QS} |



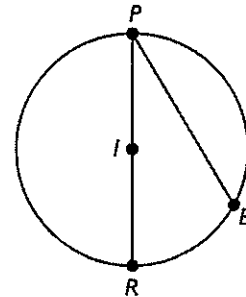
For 1–4, identify each part of the circle shown at the right.

1. the center

2. a radius

3. a chord

4. a diameter



Draw a circle with a radius of 4 cm. Then draw and label the following.

5. center *A*

6. radius \overline{GA}

7. chord \overline{DF}

8. diameter \overline{BC}

Name _____

Estimate Circumference

A line segment with both endpoints on the circle that passes through the center is called the diameter. The **circumference** is the distance around the circle.

Use a compass and a ruler to draw a circle with a radius of 5 cm. Estimate the circumference of the circle by using a string and a ruler.

Step 1 Draw the circle and mark the center. Draw a diameter through the center of the circle.

Step 2 Measure the diameter to the nearest centimeter. Record your measurement. (10 cm)

Step 3 Lay the string around the circle. Mark the string where it meets itself.

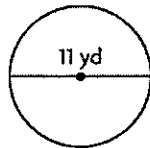
Step 4 Use the ruler to measure the string from its end to the mark you made. Measure to the nearest centimeter. Record your measurement. (30 cm)

Step 5 Use a calculator to divide the circumference of your circle by the diameter. Record your result. (3)

So, to estimate the circumference of a circle, multiply the diameter by 3.

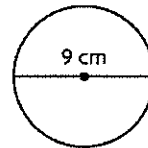
Estimate the circumference of the circle.

1.



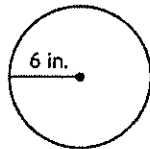
$$C \approx 3 \times \underline{\hspace{2cm}} \approx \underline{\hspace{2cm}}$$

2.



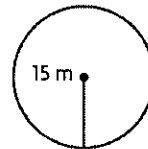
$$\underline{\hspace{2cm}}$$

3.



$$\underline{\hspace{2cm}}$$

4.



$$\underline{\hspace{2cm}}$$

Name _____

Samples and Surveys

A **survey** is a method of gathering information. Surveys are usually made of up questions. You can survey a population, which is the whole group. If the population is very large, you can survey a **sample**, or a part of the population.

| | | |
|-----------------------------|--|--|
| Random Sampling | Everyone has an equal chance of being chosen. | The names of all students in a class are placed in a hat, and one is randomly chosen. This represents random sampling because all students have an equal chance of being chosen. |
| Convenience Sampling | Individuals that are readily available are chosen. | The people standing in line at an ice cream shop are asked about their favorite ice cream flavors. This represents convenience sampling because the people in line at the same time were easily available to survey. |
| Systematic Sampling | A random individual is chosen, and then every 3rd, 4th, 5th person, etc. | Andrea surveys the 3rd person on a phone list, and then every other person. This represents systematic sampling because a pattern was used to choose people. |

Identify the sampling method.

- Sam works at an appliance store. He surveys the second customer of the day, and then every 10th customer after that.

- Trudy wants to know what movies the students in her school like to watch. She asks the students in her math class.

- Makayla randomly chooses 30 people from a list of names and asks their favorite color.

- Kai wants to know what his classmates typically eat for breakfast. He asked the students on the math team what they had for breakfast today.

Name _____

Make Predictions from Samples

When a small sample is representative of a larger sample, you can use equivalent ratios to make predictions.

Gordon took a test with 20 questions and answered 16 of them correctly. If Gordon takes a test with 110 questions, how many is he likely to answer correctly?

Step 1 Write ratios comparing the number of questions he answered correctly to the total number of questions.

$$\frac{16}{20} = \frac{\square}{110}$$

Step 2 Since 110 is not a multiple of 20, write the first ratio as a unit rate.

$$\frac{16 \div \boxed{20}}{20 \div 20} = \frac{\boxed{0.8}}{110}$$

$$\frac{0.8}{1} = \frac{\square}{110}$$

Step 3 Write an equivalent ratio by multiplying the numerator and denominator by the same value.

$$\frac{0.8 \cdot \boxed{110}}{1 \cdot \boxed{110}} = \frac{\boxed{88}}{110}$$

Since $1 \times 110 = 110$, multiply 0.8 and 1 by 110.

So, on a test with 110 questions, Gordon is likely to answer 88 of them correctly.

Solve.

- A teacher surveyed some students and found that 4 out of 8 students bring a lunch to school. If the teacher surveyed 90 students, how many would you expect to bring a lunch?

- Twenty-four people played a game at a fair, and 4 people won a prize. If 150 people play the game, how many will most likely win a prize?

- At Bayside Middle School, 4 out of 12 students are sick. In a class of 33 students, how many would you expect to be sick?

- Morton surveyed 14 classmates and found that 8 of them are voting for Amanda for class president. If Morton surveys 119 classmates, how many of the students would you expect to NOT vote for Amanda?

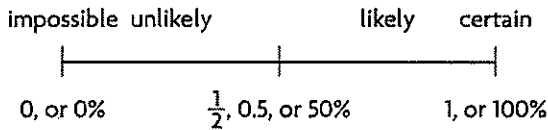
Name _____

Probability and Likelihood

An **experiment** is an activity involving chance where the results are observed or measured, such as flipping a coin. An **outcome** is a possible result of an experiment. The **sample space** of an experiment includes all possible outcomes.

Probability measures the likelihood of certain events. The number line shows that a probability of 0 means an event will not happen.

- 0 is impossible.
- 0.5 is as likely as not.
- 1 is certain.



A spinner has 5 equal-sized sections numbered 1–5. The spinner is spun and lands on a number less than 5. Describe the event as *impossible, unlikely, as likely as not, likely, or certain.*

| | |
|--|--|
| <p>Step 1 Describe the sample space.</p> | <p>The spinner can land on 1, 2, 3, 4, or 5.</p> |
| <p>Step 2 Compare the ways the event will happen to the ways that it will not happen.</p> | <p>Spinning a number less than 5 involves 1, 2, 3, or 4. Not spinning a number less than 5 involves 5. There are more ways to spin a number less than 5.</p> |
| <p>Step 3 Describe the event.</p> | <p>There are more ways to spin a number less than 5, so the event is likely.</p> |

Describe the event as *impossible, unlikely, as likely as not, likely, or certain.*

- | | | |
|--|---|---|
| <p>1. Shauna rolls a number cube labeled 1 to 6 and gets an even number.</p> <p>_____</p> | <p>2. Marc rolls a number cube labeled 1 to 6 and gets a 2.</p> <p>_____</p> | <p>3. Ling rolls 1–6 on a number cube labeled 1 to 6.</p> <p>_____</p> |
|--|---|---|

Name _____

Write Probabilities

Probability is a number that represents the likelihood of an event. You can write the probability of an event as a fraction, decimal, or percent. If an event has a probability of 0, it is impossible. If an event has a probability of 1, it is certain to happen. A probability of 50% is the same as a probability of 0.5 or $\frac{1}{2}$.

Diane is attempting free throws on a basketball court. The probability that she makes a successful attempt is 45%. Write this probability as a decimal and a fraction.

Step 1 Write the probability as a decimal.

To rename a percent as a decimal, divide the number by 100 and remove the percent sign. To do this, move the decimal left two places.

$$45\% = 0.45$$

Step 2 Write the probability as a fraction.

Write the percent as an amount out of 100, and then simplify.

$$45\% = \frac{45}{100} = \frac{9}{20}$$

Write the probability in two different ways.

1. At a factory, the probability that Omar finds a defective guitar is 0.11.

2. The probability that someone wins a prize at a carnival game is 58%.

Percent: _____

Decimal: _____

Fraction: _____

Fraction: _____

3. The probability that the Go-Getters win their next game is $\frac{7}{10}$.

4. Leyla has a 48% chance of winning the election.

Decimal: _____

Decimal: _____

Percent: _____

Fraction: _____

Name _____

Experimental Probability

The **experimental probability** of an event is a ratio that compares the number of times an event occurs to the total number of trials. Experimental probability is based on events that have already happened.

The table shows the drinks that 40 students ordered for lunch today. Use the table to find the experimental probability that a student orders chocolate milk. Write the probability as a fraction, decimal, and percent.

| | Milk | Chocolate Milk | Juice |
|--|------|----------------|-------|
| | 10 | 25 | 5 |

Step 1 Write a ratio comparing the number of students who ordered chocolate milk to the total number of students. Simplify, if possible.

$$\frac{25}{40} = \frac{5}{8}$$

Step 2 Divide the numerator by the denominator to write the probability as a decimal.

$$\begin{array}{r} 0.625 \\ 8 \overline{)5.000} \\ \underline{48} \\ 20 \\ \underline{16} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

Step 3 Write the decimal as a percent. Multiply the decimal by 100, and write a percent sign at the end.

$$0.625 = 62.5\%$$

So, the probability that a student orders chocolate milk is $\frac{5}{8}$, 0.625, or 62.5%.

Write the experimental probability as a fraction, decimal, and percent.

- Jacob plays basketball. He attempted a free throw 32 times, and made 24 shots. What is the probability that he makes a free throw?
- Marianne plays an online word game with her friend. She plays the game 8 times and wins 7 times. What is the probability that Marianne wins a game?
